Test 5 : Friday 1st July

Logarithms

This assessment contributes 6% towards the final year mark.

45 minutes are allocated for this test.

No notes or calculators of ANY nature are permitted.

Full marks may not be awarded to correct answers unless sufficient justification is given.

Name :

Non-Calculator

45 minutes

Total = $\frac{}{48}$

Do NOT turn over this page until you are instructed to do so.

Question 1 **(4 marks)**

Evaluate:

(a) $\log_2 16$

(b) $\log_3 \frac{1}{9}$

(c) $\log_e 1$

(d) $\log_9 27$

Question 2 **(4 marks)**

Solve $\log(x) + \log(x - 3) = 1$.

Question 3 **(6 marks)**

Given that $\log_a 3 = x$ and $\log_a 5 = y$,

(a) write expressions, in terms of *x* and *y*, for:

(i) $\log_a 0.6$ (2 marks)

(ii) $\log_a 45$. (2 marks)

(b) Evaluate exactly a^{4x} . (2 marks)

Question 4 **(9 marks)**

Solve the following exactly using natural logarithms.

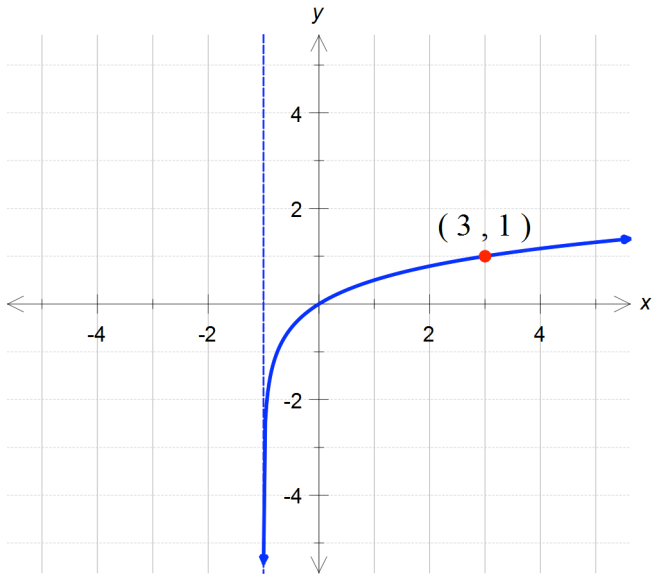
(a) $4^x = 28$ (2 marks)

(b) $5^x = 7^{x+2}$ (3 marks)

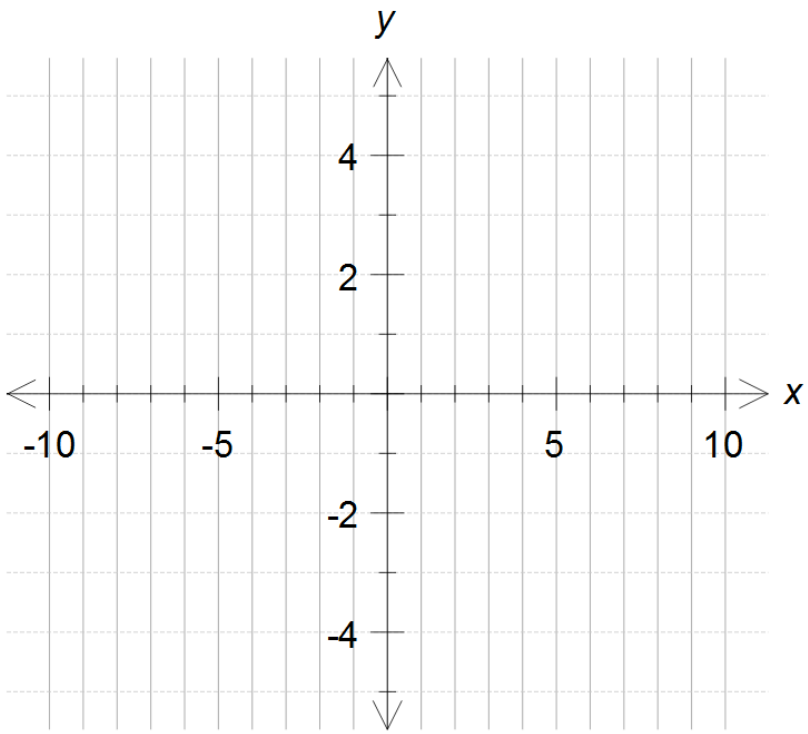
(c) $16^x - 2(4^x) - 3 = 0$ (4 marks)

Question 5 **(5 marks)**

(a) State the equation of the graph below. (2 marks)



(b) Sketch the graph of $y = -\log_2(x) + 2$ on the axes below. (3 marks)
Clearly label any key features.

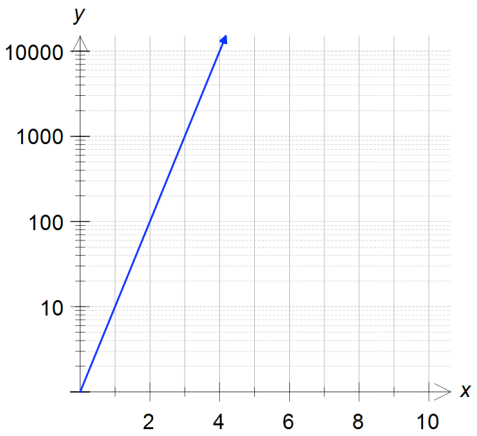


Question 6 **(4 marks)**

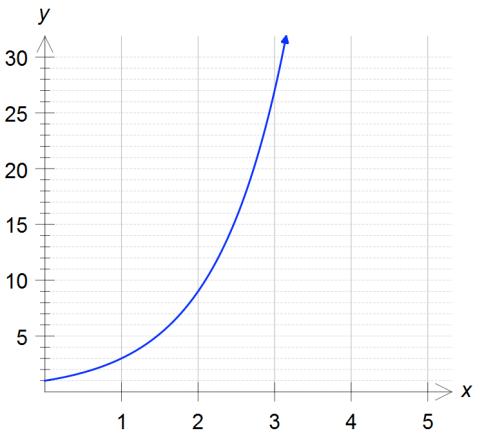
Match each graph below to one of the following equations:

A. $y = 3^x$	B. $y = 10x$
C. $y = \ln(x - 1)$	D. $y = 10^x$
E. $y = \log_4(x - 1)$	F. $y = \log_3(x)$
G. $y = \log_3(x - 1)$	H. $y = \ln(x)$

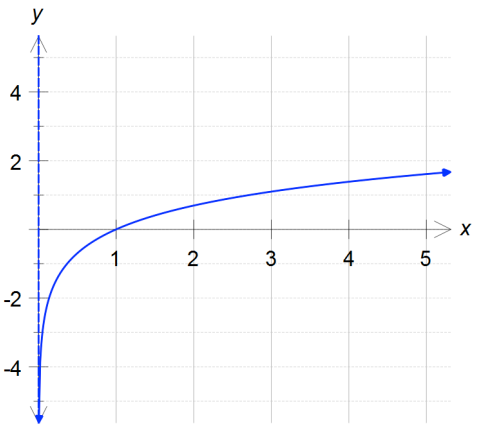
Graph i equation:_____



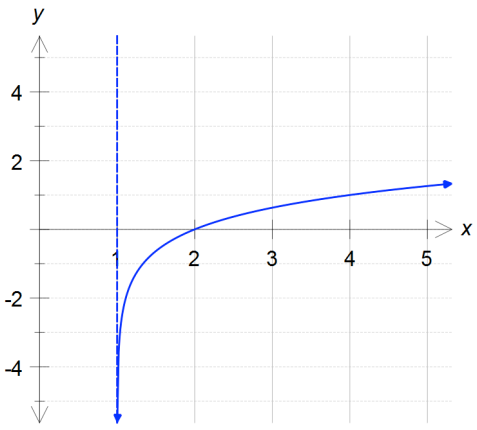
Graph ii equation:_____



Graph iii equation:_____



Graph iv equation:_____



Question 7 **(8 marks)**

(a) Differentiate the following. Do ***not*** simplify.

(i) $y = \ln(x^2 - 3x)\sin(x)$ (3 marks)

(ii) $y = 4\log_7 x$ (2 marks)

(b) Determine $\int \frac{e^{2x}}{e^{2x} + 3}\, dx$ (3 marks)

Question 8 **(4 marks)**

The approximate apparent magnitudes of two heavenly bodies are listed in the table below:

Heavenly body	Apparent magnitude m
Sirius	–1.5
Antares	1

The ratio of brightness (or intensity) $\frac{\mathrm{I_A}}{\mathrm{I}_B}$ of two objects A and B, of apparent magnitudes m_A and m_B respectively, satisfies the equation

$$\log_e\left(\frac{\mathrm{I_A}}{\mathrm{I}_B}\right) = m_B - m_A$$

(a) Determine the ratio of brightness of Sirius to Antares, stating your answer exactly. (2 marks)

(b) If the ratio $\frac{\mathrm{I}_{Jupiter}}{\mathrm{I}_{Sirius}}$ is $\sqrt{e}$, determine the apparent magnitude of Jupiter. (2 marks)

Question 9 **(4 marks)**

The position, x, of a particle at time t is given by the equation:

$$x(t) = t + \ln(t - 3).$$

(a) Determine the velocity function for the particle. (1 mark)

(b) Does the particle ever stop moving? Justify your answer. (3 marks)